

Homework Assignment 2: Due Thursday September 21

Trajectory Tracking with Linear Quadratic control.

This problem treats a variant of the linear quadratic control problem where the goal is to accurately follow a given trajectory rather than keep the system state close to zero. For a fixed sequence $y_0, \dots, y_N \in \mathbb{R}^n$ and symmetric $Q \succ 0$ and $R \succ 0$, consider the problem of minimizing the cumulative expected cost

$$\mathbb{E} \left[\sum_{k=0}^{N-1} \left((x_k - y_k)^\top Q (x_k - y_k) + u_k^\top R u_k \right) + (x_N - y_N)^\top Q (x_N - y_N) \right],$$

subject to the linear dynamics

$$x_{k+1} = Ax_k + Bu_k + w_k \quad k = 0, 1, \dots, N-1.$$

In this problem, we show that the optimal cost-to-go functions have the form

$$J_t^*(x) = x^\top K_t x + 2q_t^\top x + r_t$$

and the optimal policy has the form $\pi^* = (\mu_0^*, \dots, \mu_{N-1}^*)$ where

$$\mu_t^*(x) = L_t x + m_t.$$

That is, the optimal policy is an affine function of state.

Inventory Control with Forecasts.

Do problem 3.3 parts (a) and (b) from Bertsekas Volume I. Formulate the problem precisely (i.e. clearly specify the cost functions, state dynamics etc.). It is helpful to refer to section 1.4, pages 40-41.